Producers bargaining over a quality standard

Revision of SSE/EFI Working Paper No 618 (December 2005)

Cédric Argenton
Stockholm School of Economics*

May 8, 2006

Abstract

We study an asymmetric information model in which two firms are active on a market where buyers only observe the average quality supplied. Quantities and cost structures are exogenously given and firms compete in quality. Before choosing their qualities, they bargain over a perfectly enforcable minimum quality standard. The bargaining outcome is given by the Kalai-Smorodinsky (KS) solution. Agreement on a binding standard is possible only if the firms are sufficiently similar with respect to their production costs. The agreed-upon standard always falls short of the joint-profit-maximizing (or, for that matter, the efficient) level. It is decreasing in the high-cost producer's cost of production. Yet, it first increases then decreases with the low-cost producer's cost of production, showing that the latter's bargaining position can be enhanced by seemingly adverse cost changes.

KEYWORDS: asymmetric information, minimum quality standard, duopoly, bargaining, free riding.

JEL CODES: D43, D82, L13, L15.

---

*Box 6501, SE-113 83 Stockholm, Sweden; e-mail: cedric.argenton@hhs.se . I sincerely thank the Knut and Alice Wallenberg Foundation (Knut och Alice Wallenbergs Stifelse) for financial support. I started working on this piece of research while attending Boston University, whose funding is also gratefully acknowledged. My thesis supervisor, Jörgen Weibull, provided enormous guidance and encouragement. The paper has benefited from helpful comments from Avinash Dixit, Tore Ellingsen, Chloé Le Coq, Lars Ljungqvist, Stefan Napel, Mark Voorneveld and seminar participants at the Stockholm School of Economics. The remaining errors and shortcomings are mine only.

1 Introduction

Since Akerlof (1970) put into light the negative consequences of asymmetric information on market outcomes in his famous article on "lemons", a great deal of attention has been devoted to the study of potential remedies. Among those, the imposition of a minimal quality standard (MQS) has very often looked like a minimally invasive yet promising form of government intervention.

Leland (1979) offered an elegant formalization of these ideas. Using a continuum of price-taking producers, he showed that quality deterioration indeed takes place on unregulated markets with asymmetric information and that there are a number of instances where a MQS, although generally not first-best, increases social welfare. Leland also showed that, if the standard is set by the producers so as to maximize their joint profits, then it is in general likely, and certain if a typical Spencian (1975) condition is satisfied, that the standard will be "too high" (that is, higher than the welfare-maximizing standard), as producers have the additional monopoly incentive to decrease output so as to drive prices up.

Later on, the issue of the desirability of a MQS was preferrably addressed within the frame of a full-information, vertically-differentiated duopoly à la Mussa-Rosen (1978) where firms first choose qualities and then compete for customers. Ronnen (1991) proved that when firms compete in prices in the second stage and quality affects fixed costs, the introduction of a MQS leads to a narrowing of the quality gap that, because it increases price competition, increases welfare through higher average quality, consumer participation and consumer surplus. This contribution spurred a lot of effort aimed at checking whether this favorable effect of introducing a MQS was robust to the assumed cost structure (Crampes and Hollander [1995]), the possibility of collusion (Ecchia and Lambertini [1997]), the duopoly setting (Scarpa [1998]) or the nature of competition (Valletti [2000]). At a general level, it is possible to say that there exist some instances in which the rise in consumer surplus following the introduction of a MQS is large enough to compensate for the decrease in firm's profits so that total welfare goes up. Nonetheless, in many of these instances, one has to trade the welfare of some consumers (typically, the high-quality buyers) for the welfare of some others (typically, the low-quality buyers).

In our opinion, what has been lost in this last strand of literature was Leland's original consideration of a biased standard-setting process. Nothing indeed guarantees that a benevolent decision-maker will strive to achieve efficiency. Very often, the government, or the standard-setting organization, is not independent of producers, or insensitive to their interests. As a matter of fact, as in the case of technological standards, producers can sometimes choose which authority will certify their compliance with a norm and may therefore engage into "strategic forum shopping" (see Lerner and Tirole [2004]). In addition, there exist many professions whose regulation in general, and quality regulation in particular, is left to "representative bodies". Medicine is a good example of such an auto-regulated industry and there is a long history of suspicion toward the way licensing and other quality requirements are used by medical organizations (for an early and strong statement in the US context, see Kessel [1958]).

The key to Leland's result about the "overprovision of quality" when a MQS is set by the industry is that side-payments can be made among producers. Indeed, in his model, producers succeed in rising quality by eliminating the lowest qualities' suppliers. If profits are transferable, it is possible to compensate the producers evicted from the market. Thus, in the event these producers have a say in the standard-setting process, it is always possible to buy their approval. There are a number of instances, though, where it does not seem appropriate to assume that the required side-payments are possible. Often, especially in oligopolistic markets, antitrust considerations lead to the prohibition of direct payments between firms and to restrictions to the use of hidden-payment vehicles such as joint R&D or marketing efforts. Even the clearest cases of collusion rarely involve direct profit sharing but rather agreements on a scheme to fix prices, allot market shares or coordinate auction bids. Hence, we believe that there is some justification for studying the outcome of a process in which the incumbent suppliers bargain over the choice of a MQS as a result of their inability perfectly to redistribute the cartel profits. This question is not usefully addressed within the framework of the full-information models mentioned above. Indeed, as Crampes and Hollander (1995) have proved, in these models the high-quality firm always loses, and the low-quality firm always benefits, from the imposition of a mildly restrictive MQS. (Exit occurs if the standard is severely restrictive.) Hence, the interests of the firms radically diverge when it comes to adopting a common norm and no common ground can be found. This degeneracy of the bargaining problem does not arise when firms have a common interest in sustaining quality. This element is present under imperfect information whenever consumers care about some measure of the average level of quality in the market.

As a first attempt at tackling this research program, we construct a simple Akerlovian model of MQS bargaining between two firms. Consumers cannot observe or infer the quality of the goods or services produced by a given firm. Instead, demand depends upon the average quality of goods available in the market. Firms are free to choose the quality of their product as long as they do not violate the MQS they might have agreed upon in the first place. They differ in their marginal cost of production given quality. Once qualities have been chosen, the goods are brought to the market, the price set so as to equate supply with demand, and the profits realized. Given these profit opportunities, the Kalai-Smorodinsky (KS) bargaining solution is assumed to capture the outcome of the standard-setting negotiation between the two firms taking place ahead of production.[1]

The unregulated market is characterized by underprovision of quality and free riding on the part of the high-cost producer. We show that the bargaining problem is non-degenerate only if the firms are not too dissimilar with respect to the cost of quality. Under large cost heterogeneity, the high-cost firm does not find it profitable to agree to any quality norm. This is because it cannot profit from a standard that does not force the low-cost firm into raising quality as well. If there is a big difference in costs, it simply does not pay for the high-cost producer to undertake the "jump" in quality needed for this to happen. By contrast, if the firms' costs are not too dissimilar, then there exists a range of mutually profitable standards. Through bargaining, the firms settle for a standard that is too low, when compared to the profit-maximizing, or for that matter the (second-most) efficient, level.

At a conceptual level, these results can be attributed to the non-transferable nature of profit in our model. Joint-profit maximization would require firms to set the MQS at a level that considerably enhances the low-cost producer's profitability relatively to the high-cost producer's. This extremely unequal allocation of profits cannot arise through bargaining because there exist other MQS levels that are more favorable to the high-cost producer and these are the source of its bargaining power.

The agreed-upon MQS often exhibits the intuitive property that an increase in one firm's cost of quality leads to a decrease in the adopted quality

[1] For a characterization of the solution, see Kalai and Smorodinsky (1975). These authors attributed the first mention of this solution to Raiffa. Hence, it is sometimes described as the "Raiffa solution".

standard. Perhaps surprisingly, the converse is true when firms' costs are relatively dissimilar (yet similar enough for the bargaining problem not to be degenerate). In that case, an increase in the efficient producer's cost of quality leads to an *increase* in the adopted standard. This is because, although the high-cost producer's best profit opportunity from adopting a common standard is little affected by the increase in cost, the low-cost producer's maximal gain increases enormously, as the inefficient producer is suddenly in the position to agree to a much larger range of standards. Thus, the reduced dissimilarity between the firms opens up the range of mutually beneficial bargains in a way that is biased toward the low-cost producer. As the KS bargaining solution is monotone in the bargainers' maximal utility gains, that translates into a shift of the solution towards the low-cost producer's interests, which can be achieved only through an increase in the adopted standard.

One can question our choice of the Kalai-Smorodinsky bargaining solution. Numerous bargaining solutions have been proposed in the literature over the years and they all come with different characteristic properties or non-cooperative foundations. The KS, along with the Nash and the egalitarian solutions, are the ones that stand most of the tests that one could possibly devise (see the presentation by Thomson [1994]). One reason for our choice of the KS solution is that in this model, it leads to a tractable quadratic equation, whereas the Nash bargaining solution leads to an unappealing quartic equation. In addition, in the context of standard setting, we find the monotonicity property that differentiates the KS from the Nash solution appealing. This property requires that an expansion of the feasible set in a direction favorable to a particular agent always benefits him. This is clearly desirable as in most cases, firms must devote some energy and resources to convincing the standard-setter, be it a government agency or an assembly of producers, that the norm ought to be set at the level they favor. Like in rent-seeking models, the amount that firms are willing to spend on successful lobbying effort is equal to their potential gain. If one believes that the eventually-adopted standard is a reflection of these lobbying efforts, then monotonicity in maximal profit changes makes for a very defensible assumption.

Our model also assumes that firms produce fixed quantities. This is analytically convenient and facilitates the comparison with Leland, for in his model firms take the market price as given and decide whether or not they want to supply a pre-determined quantity. Sophisticated firms could of course realize that their quantity choice affects the market price. These strategic effects are quite complicated and we prefer overlooking them in the present study.[2] We think of the situation as one in which bargaining and production specification takes place well before the choice of output level, or one in which heavy investment in capacities is required previous to any choice concerning product characteristics.

The remainder of this article is organized as follows. In Section 2, we describe the basic model. Some preliminaries concerning alternative market structures and welfare comparisons are developed in Section 3. Section 4 presents the main results. Section 5 presents some extensions of the basic model. Section 6 concludes and suggests the directions in which the model could be taken. All the formal proofs are collected in a final section.

2 Model

Two firms, indexed by $i \in \{L, H\}$, each produce a fixed quantity α_i (set at half a unit for most of this paper) of a variant of a given good, whose quality $x_i \geq 0$ they choose. Firms differ with respect to the cost of production at a given quality level. Consumers cannot observe the quality of the individual products and cannot distinguish their origin (i.e. know the identity of their producer). Instead, consumers' demand depends upon their expectation or observation of the prevailing average quality. By assumption, the market price is set so as to equate demand with the fixed aggregate supply (which we always take to equal one unit).

We model the situation as a two-player two-stage game.[3] In Stage 1, the two firms bargain over a common minimal level of quality, denoted $m \geq 0$. In the absence of agreement $m = 0$, that is, no standard is enforced.[4] The

[2] We have given a try at analyzing a quantity-setting game in the same demand environment as this model's. See Chapter 3 of this dissertation.

[3] Because our first "stage" involves a cooperative bargaining solution, our model does not fit the technical definition of a "game" in canonical non-cooperative game theory. It would be possible to cast it into the usual framework by artificially introducing a third player called upon choosing the common standard in Stage 1, whose objective function would involve costly departures from the Kalai-Smorodinsky ratio of firms' profits. Alternatively, one can see the analysis of firms' behavior in Stage 2 as background work for the determination of the bargaining set and take the situation to be a pure bargaining problem. We find the reference to stages useful and will hence continue using it.

[4] Any strictly positive but ineffective standard (one that would not affect firms' behavior

outcome of the negotiation process is captured by the Kalai-Smorodinsky bargaining solution. The KS solution chooses the point on the Pareto frontier of the bargaining set at which the ratio of firms' actual gains over the disagreement outcome equals the ratio of their "ideal" gains, i.e. the gains they can expect under the most favorable negotiation outcome.

In Stage 2, firms take the minimum level of quality agreed upon in Stage 1 as given, and simultaneously choose the quality levels, x_L and x_H. The minimal level of quality, m, is assumed to be strictly and costlessly enforced: firms cannot choose a quality level falling short of that standard.

There is a cost associated with increasing quality. We assume that the cost per unit produced is a quadratic function of quality[5] given by $\theta_i \cdot (x_i)^2/2$, where $\theta_i \geq 0$ is a parameter standing for firm i's *cost of quality.* There are no fixed costs. Hence, each firm's total production cost is equal to $\alpha_i \cdot \theta_i \cdot (x_i)^2/2$. We assume that $\theta_H \geq \theta_L$ and it is understood that in the case when this inequality strictly holds and firms produce the same quality level, firm L enjoys a lower marginal cost than firm H, justifying our choice of subscripts.

After both firms have determined their quality levels, consumers observe (or infer) the average level of quality $\bar{x}$. Their aggregate demand is an affine function of this average. More precisely, the quantity demanded, at any given price p, is taken to be

$$D(p) = 1 + a + \bar{x} - p, \tag{1}$$

where $a \geq 0$ is a demand-shifting parameter introduced to guarantee full market coverage. This demand could arise from a continuum of consumers with valuations of the good in question uniformly distributed between 0 and $1 + a + \bar{x}$.[6]

in Stage 2) could be equivalently chosen as the disagreement point.

[5]The quadratic specification for marginal cost seems to be the most economical specification to address the problem at hand. In the case where marginal cost is a linear function of quality, there is no equilibrium in pure and interior strategies to the quality "subgame" in general, and this is so independently of the choice of the inverse demand function. There do exist "corner-solution" equilibria but they are uninteresting as they give rise to trivial bargaining problems (degeneracy or immediate agreement on a unique Pareto-efficient outcome).

[6]For instance, consumers could all be willing to buy one unit of the good only. Their preferences could be defined over that good and all the other goods they possibly care about. Absent any strong income effect, their indirect utility function could then be taken to be linearly separable in the gross utility derived from that unit and its price. The utility from consuming the good could comprise a "baseline" utility level, differing across consumers, and a valuation-of-quality term, identical across consumers. For example, a

The equilibrium market price p^* is defined by equating demand with the fixed supply, giving $p^* = a + \bar{x}$. In most of this article, we will consider the situation where both firms produce half a unit. Thus, the market price, as a function of the firms' quality levels, is

$$p^*(x_L, x_H) = a + \frac{1}{2}(x_L + x_H). \tag{2}$$

Firms strive to maximize their profits, $\Pi_i = \left[p^*(x_L, x_H) - (\theta_i \cdot (x_i)^2)/2\right]/2$. Hence, viewed as players in stage 2, their payoff functions are

$$\pi_i(x_H, x_L) = \frac{1}{2}\left[\frac{x_L + x_H}{2} - \frac{1}{2}\theta_i \cdot (x_i)^2 + a\right]. \tag{3}$$

3 Preliminaries

Before analyzing the model, it is informative to analyze some related market structures and make some welfare comparisons.

3.1 Monopoly

Consider first the monopoly case. Suppose then that there is a single decision-maker who sets x_L and x_H so as to maximize $\pi_L(x_H, x_L) + \pi_H(x_H, x_L)$, the total profit to a corporation owning both production plants or "profit centers." Implicit in the specification of this objective function is the assumption that the difference in costs across plants is not due to a difference in technologies—potentially eliminated by a merger—but to some other cause(s), for instance, heterogeneity in the quality or price of local inputs (such as land in agriculture, metals in the industry, or labor in general).

It is easy to establish that the first-order conditions for profit maximization are

$$x_L^M = \frac{1}{\theta_L} \qquad x_H^M = \frac{1}{\theta_H}, \tag{4}$$

resulting in monopoly profit

$$\Pi^M = \frac{1}{4\theta_L} + \frac{1}{4\theta_H} + a. \tag{5}$$

unit mass of consumers indexed by $t \in [0, 1]$ could have preferences represented by the utility functions $U_t = b_t + x - p$ where each consumer t derives a baseline utility from consuming one unit of the good equal to $a + t$ and a quality-related utility equal to x.

The higher the cost parameters, the lower are the qualities, as well as the monopolist's profit.

Using the familiar criterion of total surplus, welfare in our model can be expressed as

$$W(x_H, x_L) = \frac{1}{2} + a + \frac{x_L + x_H}{2} - \frac{1}{4}\theta_L \cdot (x_H)^2 - \frac{1}{4}\theta_H \cdot (x_L)^2 . \qquad (6)$$

The unrestricted maximizers of this function happen to be x_L^M and x_H^M, so that a monopoly achieves economic efficiency. The same is true if the decision-maker is restricted to choose only one quality to be produced by both plants. In that case, we have

$$\Pi^M(x, x) = x - \frac{1}{4}(\theta_L + \theta_H)x^2 + a, \qquad (7)$$

which is maximized at

$$x^M = \frac{2}{\theta_H + \theta_L}, \qquad (8)$$

the harmonic mean of x_L^M and x_H^M, just as $W(x, x)$ is. There is nothing surprising in these results. We know from Spence (1975) that the source of the divergence between the socially optimal quality and the one chosen by a profit-maximizer lies in the difference between the marginal consumer's valuation of quality improvements and the average consumer's valuation of quality improvements (where the average is taken over all infra-marginal consumers). This discrepancy does not materialize when demand is linear in quality, i.e. when consumers' marginal valuations of quality are identical, as here.

3.2 Perfect information: monopolistic competition

Another relevant case for comparison is when there are two firms but consumers *can* distinguish the two products. Then, these are sold on two separate yet interdependent markets in a monopolistically competitive manner.

If a unit mass of consumers indexed by t is uniformly distributed over $[0, 1]$ and endowed with preferences of the form $U_t = a + t + x - p$, and if they randomly patron one firm or the other when they are indifferent, then the residual demand addressed to firm L is given by

$$D_L(x_L, x_H, p_L, p_H) = \begin{cases} 1 & \text{if} \quad x_L - p_L > x_H - p_H \\ 1/2 & \text{if} \quad x_L - p_L = x_H - p_H \\ 0 & \text{if} \quad x_L - p_L < x_H - p_H \end{cases}, \qquad (9)$$

and conversely for firm H.

If firms simultaneously choose qualities and prices, then under our quantity constraint, firm i's program is

$$\max_{x_i, p_i} \quad \left[\max\left\{\frac{1}{2}, D_i\right\}\right]\left[p_i - \frac{1}{2}\theta_i \cdot (x_i)^2\right].$$

As it is clearly not optimal to price so high as to generate zero demand or to price so low as to create excess residual demand, firm i's best response always involves setting p_i equal to $x_i - x_j + p_j$. Therefore, firm i's program reduces to

$$\max_{x_i} \quad \frac{1}{2}\left[x_i - x_j + p_j - \frac{1}{2}\theta_i (x_i)^2\right],$$

which leads to

$$x_i^S = \frac{1}{\theta_i}. \tag{10}$$

Thus, there are infinitely many equilibria but, as long as the market is fully covered, they all share the features that the price differential equals the quality differential, and qualities are socially optimal. The resulting profit to each firm is of the form

$$\pi_i^S = \frac{1}{2}\left[\frac{1}{2\theta_i} + a + c\right], \tag{11}$$

where c is a function of the prices selected in a particular equilibrium. As expected, the equilibrium profit to each firm is decreasing in its own cost parameter.

3.3 Asymmetric information in the absence of standard

Now suppose that there are two firms, that consumers cannot distinguish the two products, and that there is no quality standard. Formally, this is equivalent to letting m be exogenously fixed at zero in the model outlined in Section 2.

First, observe that each firm's profit function $\pi_i(x_H, x_L)$, defined in equation (3), is continuously differentiable and strictly concave in x_i. Hence, a necessary and sufficient first-order condition for unconstrained maximization is

$$x_i^U = \frac{1}{2\theta_i}. \tag{12}$$

When firms make their decision, they do not take into account the positive externality that their effort entails for the other firm. This explains why quality is underprovided as compared to the monopoly case or the duopoly case with perfectly informed consumers.[7]

The resulting profit levels are accordingly lower:

$$\pi_i^U = \frac{1}{2}\left[\frac{1}{8\theta_i} + \frac{1}{4\theta_j} + a\right] \tag{13}$$

for $i = L, H$ and $j \neq i$. Note that firm H makes a higher profit than firm L! This reversal of the profit ranking, as compared to the separate markets case, is a consequence of the free riding occuring in this market. To see this, let us call the portion of profit that a firm can affect through its own choice of effort its *internal profit*, ι_i. One then has

$$\iota_i = \frac{1}{2}\left[\frac{x_i}{2} - \frac{1}{2}\theta_i\left(x_i\right)^2\right]. \tag{14}$$

Similarly, let us call the portion of a firm's profit that results from the impact of the other firm's effort choice on the market price its *external profit*, ε_i:

$$\varepsilon_i = \frac{1}{2}\frac{x_j}{2}, \tag{15}$$

where x_j stands for the other firm's effort choice. Then $\pi_i = \iota_i + \varepsilon_i + a/2$.

It is easily verified that in equilibrium firm L's internal profit is higher than firm H's internal profit, that is: $\iota_2 > \iota_1$. On the other hand, firm L does not benefit much from firm H's low quality choice, which contributes little to the market price, while firm H quite gains from the high quality produced by firm L, which raises the price it receives. Hence, $\varepsilon_1 > \varepsilon_2$. For the particular specification of our payoff function, with equal quantities, this latter external effect is so big as to dominate. In other terms: $\varepsilon_{1-}\varepsilon_2 > \iota_2 - \iota_1$.

4 Analysis

We now proceed to analyse the model presented in Section 2. We first consider Stage 2 (firms' production decisions after the standard has been set), which determines the boundaries of the profit possibility set defining Stage 1's bargaining problem.

[7]If the industry were composed of n firms, each selling a fraction $1/n$ of total output, the unconstrained quality choice would be $x_i^U = \frac{1}{n\theta_i}$.

4.1 Quality choice

Recall that each firm's payoff π_i is a strictly concave function of its own effort, x_i. Suppose that a standard $m \geq 0$ has been agreed upon in Stage 1, constraining firms' quality choices in Stage 2. By strict concavity, the optimal quality levels are

$$x_i^*(m) = \max\left\{\frac{1}{2\theta_i}, m\right\}. \tag{16}$$

By assumption, firm H faces a higher cost of quality. Thus, firm H always chooses a smaller quality level than firm L. From equations (3) and (16), firms' profits, $\pi_i\left(x_H^*, x_L^*\right)$ can be shown to depend on m, the minimal quality level agreed upon in Stage 1, in the following manner:

$$\begin{array}{lll}
\text{for } m < \frac{1}{2\theta_H}: & \pi_H = \frac{1}{2}\left[\frac{1}{8\theta_H} + \frac{1}{4\theta_L} + a\right] & \pi_L = \frac{1}{2}\left[\frac{1}{4\theta_H} + \frac{1}{8\theta_L} + a\right] \\
\text{for } m \in [\frac{1}{2\theta_H}, \frac{1}{2\theta L}]: & \pi_H = \frac{1}{2}\left[\frac{m}{2} + \frac{1}{4\theta_L} - \frac{1}{2}\theta_H m^2 + a\right] & \pi_L = \frac{1}{2}\left[\frac{m}{2} + \frac{1}{8\theta_L} + a\right] \\
\text{for } m > \frac{1}{2\theta_L}: & \pi_H = \frac{1}{2}\left[m - \frac{1}{2}\theta_H m^2 + a\right] & \pi_L = \frac{1}{2}\left[m - \frac{1}{2}\theta_L m^2 + a\right]
\end{array}$$

Several observations are in order. Firstly, when the standard does not constrain firms' choices, then, as explained in the previous section and as a result of free riding, the high-cost firm makes more profit than the low-cost firm.

Secondly, firm H loses from a binding quality standard, when set at an intermediate level: $m \in (\frac{1}{2\theta_H}, \frac{1}{2\theta_L}]$. This is because in this range, firm L's behavior is unaffected by the standard, whose sole effect is to force firm H to depart from its optimal choice of quality. As a consequence, π_H is decreasing in m in this interval.

Thirdly, once $m > \frac{1}{2\theta_L}$, firm L is forced to raise its quality level as well. That leads to an increase in firm H's revenues through the increase in the market price but profitability also depends on costs. Observe that, when the standard is doubly binding, firm H's profit function is single-peaked at $m = \frac{1}{\theta_H}$. Thus determining the behavior of π_H in $(\frac{1}{2\theta_L}, +\infty)$ requires us to know whether $\frac{1}{\theta_H}$ falls into that interval or not. We have $\frac{1}{\theta_H} > \frac{1}{2\theta_L}$ if and only if firm H is at a cost disadvantage relatively to firm L but this disadvantage is less than twofold. That is:

$$\theta_L \leq \theta_H < 2\theta_L. \tag{17}$$

In that case, π_H increases with m over $[\frac{1}{2\theta_L}, \frac{1}{\theta_H}]$ and decreases thereafter. Nonetheless, it is possible for π_H never to reach again its initial level. Indeed, if firm H is at a big cost disadvantage relatively to firm L, then its costs will have become very high when the standard finally reaches the zone where it affects firm L's behavior. So, even if an increase in the standard increases firm L's profit in that zone, it never brings it back to the level associated to low effort and no standard. A simple computation shows that there are gains to be reaped by firm H from the imposition of *some* effort standard only if[8]

$$\frac{2}{3} < \frac{\theta_L}{\theta_H}. \tag{18}$$

Under our assumptions, the ratio θ_L/θ_H lies between zero and one and can be interpreted as a measure of cost homogeneity, a magnitude which will play an important role in all that follows. Thus, condition (18) requires firms not to be too dissimilar in costs in order for firm H to find it profitable to put an end to free riding.

Fourthly, we observe that firm L's profit starts increasing as soon as the standard impacts firm H's behavior, that is, for all $m > \frac{1}{2\theta_H}$. There is a kink at $m = \frac{1}{2\theta_L}$, when the standard starts constraining firm L as well. Yet, π_L continues increasing with m until $m = \frac{1}{\theta_L}$, which is firm L's preferred standard.

The two diagrams below depict the firms' profits as functions of m. The thin line stands for firm H's profit while the thick line is for firm L. Figure 1 corresponds to the case where $a = 0$, $\theta_H = 1.45$ and $\theta_L = 1$. (These parameters meet condition [18] on cost homogeneity.) One can see that firm H is hurt by an intermediate-level standard but recovers once m becomes high enough. There is a range of values for which π_H is higher than the vertical intercept.

Figure 2 corresponds to the case where $a = 0$, $\theta_H = 1.9$ and $\theta_L = 1$. (These parameters do *not* meet condition [18] on cost homogeneity.) One can see that firm H is hurt by an intermediate-level standard, recovers somewhat once m becomes high enough but not so as to bring π_H back to its initial level.

[8] This condition guarantees that the maximum achieved by π_H on $[\frac{1}{2\theta_L}, \infty)$ is strictly greater than the profit achieved when $m = 0$.

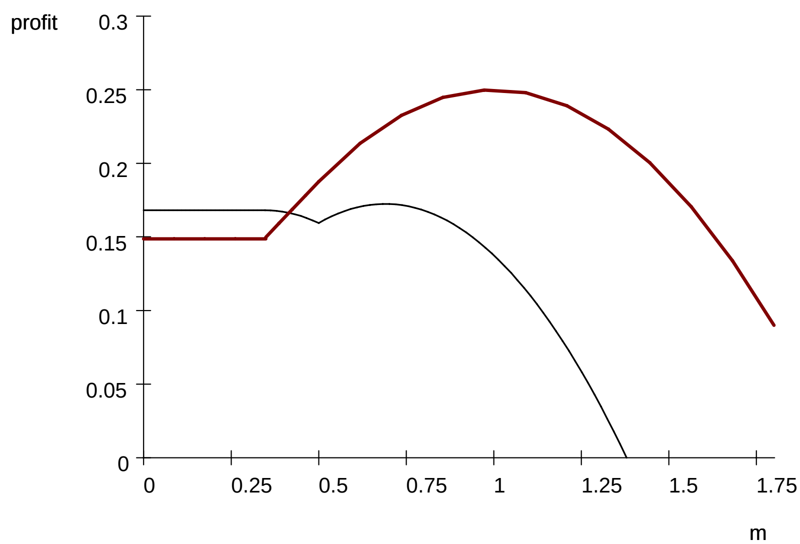

Figure 1: The two firms' equilibrium profits, as functions of the common minimum standard, m. Intermediate cost homogeneity.

4.2 Bargaining problem

A bargaining problem is usually defined with respect to a set of possible utility allocations, the *feasible set*, and a fallback or *disagreement point* in this set. Here, the disagreement point corresponds to $m = 0$ and is given by $\left(\pi_H^U, \pi_L^U\right)$, the profits to the firms when they do not adopt any standard. The table displaying firm's profits as functions of m in the previous subsection is a parametric characterization of the set of possible utility allocations in $\mathbb{R}^2$. It is well-known that in the case of the KS solution as in many others', the consideration of the set of utility changes from the disagreement allocation utilities leads to the same solution as the consideration of the original feasible set. So we can take the feasible set to be the set F of *gains* over the no-standard-case profits, and the disagreement point to be $(0, 0)$. Because the KS solution satisfies individual rationality, one is in fact only interested in the set of outcomes in which firms' gains are positive. A bargaining problem is said to be *degenerate* if the feasible set does not contain any point corresponding to strict gains over the disagreement utilities for all parties

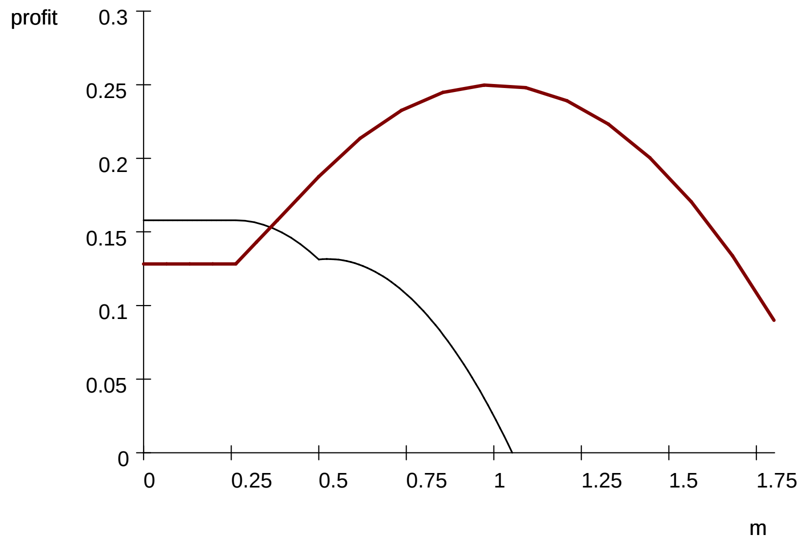

Figure 2: The two firms' equilibrium profits, as functions of the common minimum standard, m. Small cost homogeneity.

(that is, here, if $F \cap \mathbb{R}^2_{++} = \varnothing$).

Because the KS solution is defined by reference to each party's most favorable outcome, we would like to know which standards lead to the highest gains to firm H and firm L. At the same time, by individual rationality, only the outcomes in which both firms make at least as much profit than in the no-standard case really matter. We saw in the previous subsection that firm H is susceptible to make more profit than in the absence of a standard only if the MQS affects both firms and if cost homogeneity is large enough. By comparing firm H's profit under such a doubly binding standard to firm H's profit when its quality choice is unrestricted, one can find the interval $[z_1, z_2]$ of relevant standards.[9] There is no issue about firm L's gains being non-positive in that interval since its revenues are the same as firm H's but its

[9]Formally, this is done by computing the roots of the equation

$$\pi_H^U = \pi_H \left[x_H^*(m), x_L^*(m) \right].$$

costs of production are smaller by assumption. So the best feasible outcome for firm H is always $m = 1/\theta_H$. On the other hand, firm L's profit is maximized at $m = 1/\theta_L$ but that standard may or may not lie within $[z_1, z_2]$. Indeed, if cost homogeneity is not so small as to prevent any agreement but still consequential, then firm H makes a loss when $m = 1/\theta_L$, as this level of quality is simply too costly to produce. In that case (to which we will refer as the intermediate cost homogeneity case), the best outcome for firm L under the constraint that firm H does not earn less than its disagreement profit corresponds to $m = z_2$, which is smaller than $1/\theta_L$.[10] This is the situation depicted in Figure 1.

Thus, we have to distinguish three cases. Under small cost homogeneity, firm H never benefits from agreeing to a binding standard. Under intermediate cost homogeneity, firm's H favorite standard is $1/\theta_H$, while firm L's maximal feasible gain corresponds to $z_2 < 1/\theta_L$. Under large cost homogeneity, firms' maximal gains correspond to $1/\theta_H$ and $1/\theta_L$, respectively. We characterize these three cases more precisely in the following lemma.

Lemma 1 *Firms' most favorable outcomes under the condition that all parties make at least as much as under disagreement are given by the following table:*

Cost homogeneity	*Parameter range*	*firm L's best outcome*	*firm H's best outcome*
small	$0 \le \frac{\theta_L}{\theta_H} \le \frac{2}{3}$	$m = 0$	$m = 0$
intermediate	$\frac{2}{3} < \frac{\theta_L}{\theta_H} < \frac{4}{3+\sqrt{5}}$	$m = z_2 = \frac{1+\sqrt{\frac{3}{4}-\frac{1}{2}\frac{\theta_H}{\theta_L}}}{\theta_H}$	$m = \frac{1}{\theta_H}$
large	$\frac{4}{3+\sqrt{5}} \le \frac{\theta_L}{\theta_H} \le 1$	$m = \frac{1}{\theta_L}$	$m = \frac{1}{\theta_H}$

A proof of the lemma can be found in the appendix.

4.2.1 Small cost homogeneity

If condition (18) on costs is not satisfied, i.e. if the firms display a small level of cost homogeneity, then firm H never benefits from agreeing to a quality standard. Hence, the bargaining problem in Stage 1 is degenerate, as

[10] A variant of the KS solution, suggested by Kalai and Rosenthal (1978), chooses the allocation that sets players' utilities proportional to their most optimistic expectation of gain, even when this gain entails losses for one or more players, i.e. when the corresponding allocation is not individually rational. For the problem at hand, it is hard to argue that such allocations should play a role in the determination of the final bargain.

there are no "gains from trade" to be shared among the bargainers. Strictly speaking, the Kalai-Smorodinsky solution is not defined in that instance.[11] Intuition nevertheless suggests that firm H will refuse any standard above its unconstrained optimum $x_H^U = \frac{1}{2\theta_H}$ and accept any standard less than or equal to the latter, out of indifference. The limit case when $\theta_L/\theta_H = 2/3$ opens the possibility that firms agree on setting m equal to $\frac{1}{\theta_H}$, as this is profitable to firm L and makes no difference to firm H. We summarize these considerations in the following paragraph.

Remark 2 *In the case of small cost homogeneity ($\theta_L/\theta_H \leq 2/3$), the bargaining problem is degenerate: Any ineffective standard $m \leq 1/(2\theta_H)$ might be chosen. Consequently, firms' qualities and profits are the same as in the absence of a standard. If $\theta_L/\theta_H = 2/3$, then there might also be an outcome in which $m = 1/\theta_H$, in which case quality, price, and firm L's profit are higher than in the absence of a quality standard.*

4.2.2 Large cost homogeneity

Consider now the case when the firms have very similar cost parameters, that is

$$\frac{4}{3+\sqrt{5}} \leq \frac{\theta_L}{\theta_H} \leq 1. \tag{19}$$

Then, the downward-sloping section of the bargaining set frontier, which corresponds to quality standards in $\left[\frac{1}{\theta_H}, \frac{1}{\theta_L}\right]$, entirely lies to the northeast of the disagreement point. It is understood that in case negotiations break down, no binding standard will be imposed. In that case firm i's payoff will be π_i^U, its *guaranteed,* or *disagreement, profit.* Firm i's *actual gain* following the imposition of a binding standard m is defined as the difference between firm i's actual payoff and its guaranteed profit. Firm i's *maximal gain* is defined as the highest achievable profit increment under the condition that players get at least as much as their guaranteed profit.

One can picture the situation in the profit space $\pi_H \times \pi_L$. Figure 3 corresponds to $\theta_H = 1.1$, $\theta_L = 1$, $a = 0$

The point $(0,0)$ corresponds to the cases when $m \in [0, \frac{1}{2 \cdot 1.1}]$, i.e. the circumstances in which the quality standard is not binding. The beginning

[11] Neither are other bargaining solutions. Thus, there is a sense in which too big a difference in costs prevents agreement on a MQS, independently of the details, or features, of the bargaining process.

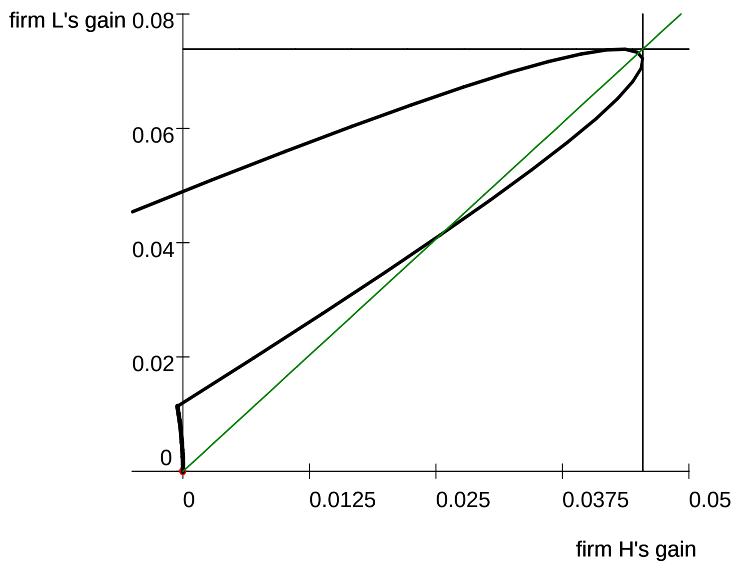

Figure 3: The feasible set and the KS solution when costs are very similar

of the line (before the kink) is for standards $m \in [\frac{1}{2 \cdot 1.1}, \frac{1}{2}]$. Recall that in this range, because quality is costly but the norm does not affect firm L's behavior, firm H's profit decreases quadratically while firm L's profit increases linearly. Thus, firm H's gain is negative, while firm L's is positive. The part of the line that follows the kink is for higher standards. Its intersections with the vertical axis correspond to $m = z_1$ and $m = z_2$. The maximal gain for firm H is achieved at $m = \frac{1}{1.1}$ (vertical dashed line). The maximal gain for firm L is achieved at $m = \frac{1}{1}$ (horizontal dashed line). The KS solution for this bargaining problem is found at the intersection of the profit frontier with the upward-sloping dashed line. At the solution one must have that the ratio of firms' actual gains equals the ratio of firms' maximal gains.

It happens that a reasonably simple closed-form solution for the agreed-upon standard $\hat{m}$ is available in that case.

Proposition 3 *If* $\frac{4}{3+\sqrt{5}} \leq \frac{\theta_L}{\theta_H} \leq 1$*, then the standard upon which firms agree* *is given by*

$$\hat{m} = \frac{5 + \sqrt{25 - 6\frac{(\theta_L+\theta_H)^2}{\theta_L\theta_H}}}{3(\theta_L + \theta_H)}. \tag{20}$$

A proof of this claim can be found in the appendix.

Note that when $\theta_H = \theta_L = \theta$, the agreed-upon standard $\hat{m}$ equals $1/\theta$. Indeed, when the two firms are exactly identical, the downward-sloping section of the bargaining set collapses to a single point, corresponding to $1/\theta$, and the feasible set reduces to a segment along the 45-degree line. The set $[z_1, z_2]$ still stands for the range of mutually profitable standards but the interests of the firms are aligned in the sense that, to the left of $1/\theta$, both prefer to increase the standard and, conversely, to the right of $1/\theta$, both prefer to decrease it. Since there exists a unique Pareto-optimal outcome, it is attained by any bargaining solution satisfying Pareto-optimality, in particular by the KS solution.

The comparative statics of the equilibrium standard are as follows.

Proposition 4 *If* $\frac{4}{3+\sqrt{5}} \leq \frac{\theta_L}{\theta_H} \leq 1$, *then (i) for any fixed* θ_L, $\hat{m}$ *is strictly decreasing in* θ_H*; (ii) for any fixed* θ_L, $\hat{m}$ *is strictly decreasing in* θ_H*.*

A proof of this claim can be found in the appendix.

These small variation effects might look intuitive but they do not necessarily follow from casual observation because in the space of profits, a change in one cost parameter displaces the frontier of the bargaining set as well as the disagreement point.[12]

In equilibrium, firm L makes a higher profit than firm H, which is a reversal of the ranking under disagreement. This outcome is to be expected

[12]For instance, if θ_H goes up, then firm H's maximal gain goes down. Indeed, at its preferred effort standard, $1/\theta_H$, its payoff decreases to a big extent, as both firms reduce the quality provided to the market. In addition, its guaranteed profit is also affected by the increase in θ_H but to a lesser extent as firm L's quality remains unchanged. At the same time, firm L's maximal gain goes up because there is no change to its payoff under its preferred standard, $1/\theta_L$, and its guaranteed profit goes down as a result of the decrease in external profit. The KS solution thus dictates an increase in the ratio of firm L's actual gain to firm H's actual gain. Precisely, at any given binding standard, firm L's actual gain is mechanically increased through the decrease in its guaranteed profit. So, a priori, it is not clear that the required decrease in the ratio will necessitate a decrease in the equilibrium standard. Our result shows that it does.

once both firms produce the same quality, since they have the same revenues but by assumption firm L has smaller costs.

A straightforward computation allows us to make the following claim.

Remark 5 *If* $\frac{4}{3+\sqrt{5}} \leq \frac{\theta_L}{\theta_H} < 1$, *then* $\hat{m} < x^M$.

That it, the standard that firms agree upon is lower than the (second-most) profit-maximizing level, which we know from Section 2 is also the (second-most) efficient level. This is a consequence of the non-transferable nature of profit in our model. Joint-profit maximization would require firm H to produce at a higher quality level but this would lead to an even greater enhancement of firm L's relative profitability that is precluded by the bargaining solution. If firm L could somehow share its profit with firm H, it would be possible to achieve efficiency.

In other terms, the initial free riding that plagues the industry in the absence of a standard, by allowing firm H to be (relatively) very profitable, empowers it too much. Indeed, since the geometric mean of two real numbers is smaller than their arithmetic means, it is clear that $\hat{m}$ lies to the left of the midpoint between $1/\theta_H$ and $1/\theta_L$. So, to the extent that the agreed-upon standard is closer to firm H 's favorite standard than to firm L's, the former can be said to have more bargaining power than the latter. To the extent that the agreed-upon standard falls short of the social optimum, it can be said to have too much of it.

This remark obviously does not apply to the case where $\theta_H = \theta_L = \theta$, in which case firms, equal in all respects, agree to implement the first-best standard $1/\theta$.

4.2.3 Intermediate cost homogeneity

Suppose now that $\frac{2}{3} < \frac{\theta_L}{\theta_H} < \frac{4}{3+\sqrt{5}}$. Then firm L's maximum feasible gain is no longer achieved at $m = \frac{1}{\theta_L}$, but at $m = z_2 = \frac{1+\sqrt{\frac{3}{4}-\frac{1}{2}\frac{\theta_H}{\theta_L}}}{\theta_H}$ as firm H would refuse any standard delivering less than its disagreement profit. One can picture the situation in the profit space $\pi_H \times \pi_L$. Figure 4 corresponds to $\theta_H = 1.4$, $\theta_L = 1$, $a = 0$.

It is easily seen that firm L's maximal gain corresponds to so high a standard that firm H would make less than its guaranteed profit. Thus, the best that firm L can reasonably expect is the standard corresponding to the

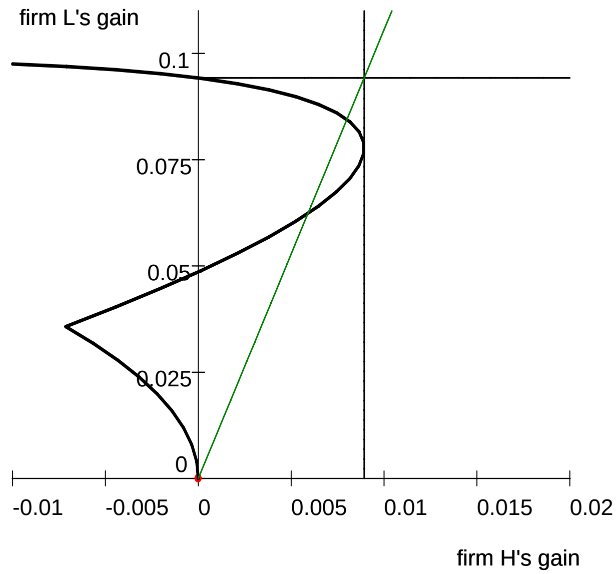

Figure 4: The feasible set and the KS solution when costs are somewhat similar

intersection of F with the vertical axis, which is z_2. Again, at the solution one must have that the ratio of firms' actual gains equals the ratio of firms' maximal gains, which translates into a quadratic equation whose bigger root, $\tilde{m}$, is the solution to our bargaining problem.

It is possible to get a closed-form expression for this root but it is not appealing, its derivatives being too complicated to be easily signed. Comparative statics can nevertheless be studied by recalling that the defining equation of the solution (displaied in the appendix) has the following form:

$$\frac{A}{B} = \frac{C}{D},$$

where A is firm L's actual gain, B Firm H's actual gain, C Firm L's maximal gain, and D firm H's maximal gain. Because of the single-peakedness of the polynomial $m - \frac{1}{2}\theta_i m^2$, A/B is strictly increasing in m on $\left[\frac{1}{\theta_H}, \frac{1}{\theta_L}\right]$. So one can start from a situation where the equality prevails, introduce a "small" change to either θ_H or θ_L, and look at the resulting change in C/D. If A/B, evaluated at the initial solution (but at the new θ_H or θ_L), has not changed to the same extent, then it must be that $\tilde{m}$ has changed in order to bring the two ratios back into equality. This way we are able to make the following claim.

Proposition 6 *If $\frac{2}{3} < \frac{\theta_L}{\theta_H} < \frac{4}{3+\sqrt{5}}$, then for a given θ_L, $\tilde{m}$ is strictly decreasing in θ_H.*

A proof of this claim can be found in the appendix.

By contrast, the changes in $\tilde{m}$ brought about by small changes in θ_L do not always go in the same direction. It is possible to show that if the homogeneity in costs between firms is quite low, then an increase in θ_L leads to an increase in the agreed-upon standard.

Proposition 7 *There exists $c \leq \frac{4}{3+\sqrt{5}}$ such that if $\frac{\theta_L}{\theta_H} \in \left(\frac{2}{3}, d\right)$, then for any given θ_H, $\tilde{m}$ is strictly increasing in θ_L.*

A proof of this claim can be found in the appendix. The argument is only sketched here.

When $\frac{\theta_L}{\theta_H}$ is very close to $\frac{2}{3}$, there is a very small interval of standards $(\frac{1}{\theta_H}, z_2)$ to the right of $\frac{1}{\theta_H}$ that firm H could agree upon as generating more than the disagreement level of profit. Because we are so close to $\frac{1}{\theta_H}$, we are very much at the top of firm H's profit hill under a doubly-binding standard (when one pictures π_H as a function of m). Any increase in θ_L, by decreasing firm H's guaranteed profit, has the effect of pushing z_2 to the right along a nearly horizontal trajectory. Thus, there is an enormous change in z_2, that increases firm L's maximal gain at an extraordinary rate. As the KS bargaining solution is monotonic in maximal utility gains over the disagreement utility, that translates into a big increase in firm L's actual gain, which is achievable only through a rise in the effort standard. The key point is that firm L's profit under its favorite standard of all, $\frac{1}{\theta_L}$, does not change much following the rise in θ_L but that does not matter as this outcome is so unfavorable to firm H that it cannot be agreed-upon anyway. By contrast, among the outcomes that firm H can rationally accept, firm L's maximal gain changes tremendously because firm H is suddenly open to a much larger range of standards. Thus, the enhanced similarity between the firms opens up the range of mutually beneficial bargains in a way that is biased towards firm L.

In fact, because the derivative of z_2 with respect to θ_L is infinite at $\frac{\theta_L}{\theta_H} = \frac{2}{3}$, so is the derivative of $\tilde{m}$. That implies that the derivative of firm L's profit function is also infinitely positive at that point. In other terms, when firm L's cost increases in a way that makes firm H suddenly willing to agree to a much broader range of standards, then firm L's profit goes up: *It is profitable to become less efficient if it is the price to pay to become more similar*. Again, this is so because the change in θ_L has a first-order effect on firm L's maximum gain.

This behavior of the agreed-upon standard when cost homogeneity is verging to smallness might raise doubt about the appropriateness of our bargaining solution: Might firm L not be tempted by masquerading as a slightly more inefficient firm, when information about costs is not perfect, in order to get a MQS closer to its favorite one? For instance, if the standard-setter is a public authority, might firm L not lie about its cost in order to fool the arbitrator? This questions are out of the scope of this paper. We note that Moulin (1984) proved that the KS solution was implementable as the unique subgame-perfect Nash equilibrium of a mechanism in which players are asked to bid "fractions of dictatorship."[13]

It is readily observed that, were we to compute the level of the standard that equated firms' actual gains ratio to firms' ideal gains ratio, disregarding individual rationality considerations, the algebraic expression would be given by $\hat{m}$ in Proposition 3, and we would have $\hat{m} > \tilde{m}$. (The dotted ray would rotate counterclockwise in Figure 4.) From Remark 5 we know that $\hat{m}$ is smaller than the (second-most) efficient standard. As a result, we can draw a similar conclusion for $\tilde{m}$.

Remark 8 *For any θ_H and θ_L such that $\frac{2}{3} < \frac{\theta_L}{\theta_H} < \frac{4}{3+\sqrt{5}}$, we have $\tilde{m} < \hat{m} < x^M$.*

This implies that the adopted standard lies even closer to firm H's favorite choice than firm L's.

[13] Because of its scale invariance and symmetry properties, the KS solution in effect maximizes a Rawlsian social welfare function once the problem is suitably normalized. Thus, implementability is not surprising. For two-person problems, Kalai and Rosenthal (1978) had already offered a mechanism Nash-implementing the KS solution (among other equilibria). Myiagawa (2002) proposes a general game form that implements the KS solution (along with the Nash solution and some weighted utilitarian solutions) in subgame-perfect equilibrium.

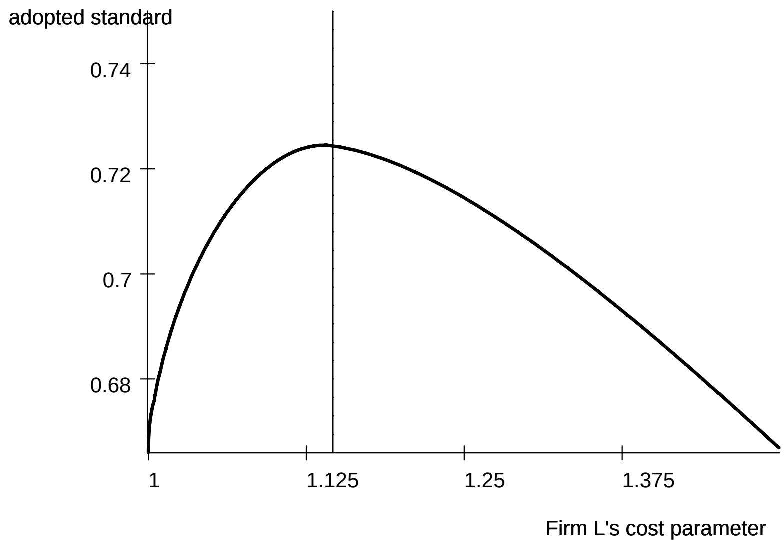

Figure 5: The relationship between the adopted standard and Firm 2's cost parameter in the case when $c_1 = 1.5$.

4.3 Numerical example

Fix θ_H at 1.5. The treshold $\frac{\theta_L}{\theta_H} = \frac{4}{3+\sqrt{5}}$ corresponds to a value of $\frac{6}{3+\sqrt{5}} \simeq 1.14$ for θ_L. Figure 5 depicts how the adopted standard behaves when firm L's cost parameter varies from 1 to 1.5 , the range for which the bargaining problem is not degenerate. Values below the treshold (to the left of the dashed vertical line on the graph) correspond the case of intermediate cost heterogeneity; values above the treshold (to the right of the dashed line) correspond to the case of small cost heterogeneity.

When firm L is very efficient (θ_L close to 1), the adopted standard is very close to the one favored by firm H, $\frac{2}{3}$, which in the limit is the only one that the latter can agree to. As firm L's cost increases, the agreed-upon standard goes up first, peaks a bit before the treshold before decreasing toward $\frac{2}{3}$, the norm that both firms happen to favor as θ_L converges to 1.5.

The graph makes clear that the derivative of the equilibrium standard right of 1 is infinitely positive, a fact used in the proof of Proposition 8, and implying that firm L's profit, as a function of θ_L, is increasing in that region.

A possible measure of firms' bargaining power is the distance between the adopted standard and their favorite standard. One could in principle construct an index of Firm L's bargaining power by expressing the distance from the adopted standard to Firm H's favorite standard as a fraction of $\left(\frac{1}{\theta_L} - \frac{1}{\theta_H}\right)$. The higher this index, the closer to Firm L's ideal point the adopted standard is. The remarks made above imply that this index is bounded above by 0.5. Indeed, in the numerical example it increases monotonically toward this value as θ_L goes up. This is again an indication that similarity is desirable in this context.

5 Extensions

5.1 Uncertainty in production

The basic model can be straightforwardly extended to deal with uncertainty in the production process making qualities random variables. Under risk-neutrality on the part of producers and consumers, it is sufficient to reinterpret the variable x_i as "effort" determining the mean of the product quality distribution and all the results carry over without modification.

5.2 Risk-aversion

The model described in Section 2 assumed that demand linearly depended upon the average quality, implying that the valuation of the good was independent of the other features of the lottery over qualities that consumers faced when purchasing the good. One might argue that consumers' willingness to pay should decrease with the uncertainty associated to the sampling procedure, as a consequence of risk-aversion. If consumers' willingness to pay is taken to correspond to the lottery's certainty equivalent, then all the qualitative features of our model are preserved.

For instance, one can assume that a unit mass of consumers indexed by t is uniformly distributed on $[0, 1]$. Consumer t's preferences can be represented by the following utility function:

$$U_t = b_t + \sqrt{x} - p, \tag{21}$$

where baseline utility, b_t, equals $a + t$. With this structure, the demand associated to a fair lottery between qualities x_L and x_H, whose arithmetic average is $\bar{x}$, can be given by[14]

$$D(p) = 1 + a + \frac{1}{2}\bar{x} + \sqrt{x_L x_H} - p. \tag{22}$$

Thus, firm i's profit in the quality subgame is

$$\pi_i(x_H, x_L) = \frac{1}{2}\left[a + \frac{1}{2}\frac{x_L + x_H}{2} + \frac{1}{2}\sqrt{x_L x_H} - \frac{1}{2}\theta_i \left(x_i\right)^2\right]. \tag{23}$$

This expression is strictly concave in x_i. Because consumers are willing to pay less when they face a large quality differential, there is now an additional incentive for firms to choose qualities that are close to each other. As a result, the optimal choice of quality is no longer independent of the other firm's decision. Nevertheless, there is a unique pure-strategy Nash equilibrium of the quality subgame in which

$$\begin{aligned} x_H^U &= \left(\tfrac{\theta_L}{\theta_H}\right)^{\frac{2}{3}} \cdot x_L^U \\ x_L^U &= \frac{1+\left(\frac{\theta_L}{\theta_H}\right)^{\frac{1}{3}}}{4\theta_L}. \end{aligned} \tag{24}$$

It is easily verified that firm H's (firm L's) quality is higher (lower) than under risk neutrality. Firm H produces only a fraction of the quality chosen by firm L. A monopolist owning both plants would keep the ratio of x_H to x_L unchanged but set

$$x_L^M(0) = \frac{1 + \left(\frac{\theta_L}{\theta_H}\right)^{\frac{1}{3}}}{2\theta_L} = 2x_L^U, \tag{25}$$

showing that free riding is as momentous under risk-aversion as under risk-neutrality. Because the marginal valuation of quality improvements is still the same across consumers, the monopolist's choices would be mimicked by

[14] Under monotonic preferences represented by the Bernoulli utility function U, the certainty equivalent (CE_L) associated to a fair lottery L between x_L and x_H is defined by $U(CE_L) = \frac{1}{2}U(x_L) + \frac{1}{2}U(x_H) \equiv EU(L)$. If $U(x) = \sqrt{x}$, then $CE_L = \frac{1}{2}\bar{x} + \frac{1}{2}\sqrt{x_L x_H}$. The same preferences can be represented by the utility function $V(L) = CE_L$ since for two lotteries L_1 and L_2, $L_1 \succsim L_2$ iff $EU(L_1) \geq EU(L_2)$, which by definition is equivalent to $U(CE_1) \geq U(CE_2)$, which by monotonicity is equivalent to $CE_1 \geq CE_2$, thus justifying our assumption that demand is linear in the certainty equivalent.

a social planner seeking to maximize total surplus. Hence, so far all the qualitative features of the model are preserved.

Observe now that once the MQS is doubly binding, i.e. when $x_L = x_H = m$, the profit function reduces to the one studied in Section 4! So there is no change to the Pareto frontier of the bargaining set and no change either to the second-most efficient MQS. The disagreement point only is affected by the introduction of risk-aversion, moving northwestwardly as a smaller quality differential makes for less initial free riding. Consequently, the agreed-upon standard is higher than under risk-neutrality as firm H's bargaining position is weakened. In turn, that implies that the adopted standard is closer to the second-best standard than under risk-neutrality.

One should be careful with the interpretation of this model. The information structure is such that consumers are in all circumstances aware of the characteristics of the lottery they face. That is, they know the individual product qualities but are unable physically to distinguish the two products at the time of purchase. The absence of labelling or branding is key here.[15]

5.3 Unequal market shares

Recall that α_H was taken to be firm H's market share. Since we maintain the assumption that aggregate supply equals one, it is also firm H's fixed quantity. For simplicity we will denote it by α and let $1-\alpha$ stand for firm L's quantity. When considering this situation, it is again essential to be very clear about the information structure characterizing the demand side of the model: here, as in the original "lemons" example, the probability to draw a variant of a given quality equals the market share of that variant.

In Stage 2, since consumers care about the weighted average of qualities, firm i's profits are given by

$$\pi_i\left(x_L, x_H\right) = \alpha_i \left[\alpha_i x_i + (1-\alpha_i)x_{-i} - \frac{1}{2}\theta_i \cdot (x_i)^2 + a\right]. \tag{26}$$

A monopolist owning both plants would again select

$$x_H^M = \frac{1}{\theta_H} \qquad x_L^M = \frac{1}{\theta_L}, \tag{27}$$

[15] The entire analysis could be conducted by assuming that $U_t = a + t + x^{\frac{1}{k}} - p$ for $k > 1$. The parameter k, because it governs the curvature of the function U, is a measure of quality risk aversion. Demand could be taken to be linear in the certainty equivalent $CE = \left[\frac{1}{2}\left(x_L\right)^{1/k} + \frac{1}{2}\left(x_H\right)^{1/k}\right]^k$, a familiar CES form.

as would a benevolent social planner. If they had to specify quality uniformly, they would both choose

$$x^M = \frac{1}{\alpha\theta_H + (1-\alpha)\theta_L}, \tag{28}$$

the weighted harmonic mean of x_H^M and x_L^M.

In the absence of a MQS, the duopolists would choose

$$x_H^U = \frac{\alpha}{\theta_H} \qquad x_L^U = \frac{1-\alpha}{\theta_L}, \tag{29}$$

which is a direct generalization of the results in Section 4. Intuitively, the downward quality distortion is caused by the positive externality associated with a quality improvement. Since this externality is proportional to market share, so is the quality distortion: the smaller firm H's market share, the easier it takes it. The associated profits are

$$\begin{aligned} \pi_H^U &= \alpha\left[\tfrac{1}{2}\tfrac{\alpha^2}{\theta_H} + \tfrac{(1-\alpha)^2}{\theta_L} + a\right] \\ \pi_L^U &= (1-\alpha)\left[\tfrac{\alpha^2}{\theta_H} + \tfrac{1}{2}\tfrac{(1-\alpha)^2}{\theta_L} + a\right] \end{aligned} \tag{30}$$

It is easy to show that for any $\theta_H > \theta_L$, there exists $d \in \left(\frac{1}{3}, \frac{1}{2}\right)$ such that $\pi_H^U > \pi_L^U$ for any $\alpha \in [d, 1]$. That is, provided it is not minuscule, firm H makes more profit than firm L in the absence of an effective standard.

Since bargaining really takes place over margins (by scale invariance of the KS solution), it is more interesting to look at the conditions under which firm H's margin, denoted by ξ_H, is greater than firm L's, denoted by ξ_L. Observe that

$$\xi_L - \xi_H = \frac{1}{2}\left[\frac{\alpha^2}{\theta_H} - \frac{(1-\alpha)^2}{\theta_L}\right], \tag{31}$$

which is strictly increasing in α. It is a matter of computation to show that

$$\xi_H \geq \xi_L \Longleftrightarrow \alpha \leq \frac{1}{1+\sqrt{\frac{\theta_L}{\theta_H}}}, \tag{32}$$

the last quantity always being greater than $1/2$.

We have to distinguish two cases here. If α is small, then $x_H^U \leq x_L^U$; that is, firm H free-rides on firm L. If α is large enough, then we are in the somewhat perverse case where the low-cost firm enjoys so low a market share that it chooses to free-ride on the high-cost firm. Our results can be straighforwardly generalized in the first case.

From (29) above, it is straightforward to derive that

$$x_H^U \leq x_L^U \Longleftrightarrow \alpha \leq \frac{1}{1+\frac{\theta_L}{\theta_H}}. \tag{33}$$

Observe that this treshold, which is always equal to, or greater than, 1/2, is always larger than $1/\left(1+\sqrt{\theta_L/\theta_H}\right)$.

We get a table of profits as functions of the MQS very similar to the one in the equal market shares' case (omitting the a-terms):

for $m < \frac{\alpha}{\theta_H}$:	$\pi_H = \alpha\left[\frac{1}{2}\frac{\alpha^2}{\theta_H} + \frac{(1-\alpha)^2}{\theta_L}\right]$	$\pi_L = (1-\alpha)\left[\frac{\alpha^2}{\theta_H} + \frac{1}{2}\frac{(1-\alpha)^2}{\theta_L}\right]$
for $m \in [\frac{\alpha}{\theta_H}, \frac{1-\alpha}{\theta_L}]$:	$\pi_H = \alpha\left[\alpha m + \frac{(1-\alpha)^2}{\theta_L} - \frac{1}{2}\theta_H m^2\right]$	$\pi_L = (1-\alpha)\left[\alpha m + \frac{1}{2}\frac{(1-\alpha)^2}{\theta_L}\right]$
for $m > \frac{1-\alpha}{\theta_L}$:	$\pi_H = \alpha\left[m - \frac{1}{2}\theta_H m^2\right]$	$\pi_L = (1-\alpha)\left[m - \frac{1}{2}\theta_L m^2\right]$

Observe again that once the standard is binding for both firms (for $m > (1-\alpha)/\theta_L$), the unit margins are exactly the same as in the equal market shares' case. Thus, there is no change to the Pareto frontier of the bargaining set; only the disagreement point is affected by the quantity asymmetry. In particular, for $\alpha \leq 1/2$, there is initially more free riding and that enhances firm H's bargaining position, leading to the adoption of a standard that is lower than in the case of equal market shares. Any decrease in α causes the disagreement point to move southeastwardly, leading to a decrease in the adopted standard (if anything).

It is possible to generalize the results concerning the role of cost homogeneity in the determination of the bargaining outcome. Firm H never benefits from a standard that affects its behavior only. It can benefit from a doubly binding standard only if θ_L/θ_H is greater than $[2(1-\alpha)]/[1+\alpha]$, which provides an upper bound for the "small homogeneity" region. In that case $1/\theta_H$ remains its favorite outcome. This condition cannot be satisfied if $\alpha < 1/3$.[16] Thus, if the market share enjoyed by the high-cost firm is too small, free riding is intense and there is no hope of getting the problem solved through a standard-setting procedure that puts any weight on firm H's profit, whatever the level of cost homogeneity. Moreoever, the bound is decreasing in α. Taken together, these observations justify the following remark.

[16] Indeed, in that case $[2(1-\alpha)]/[1+\alpha] > 1$.

Remark 9 *In the region where the high-cost firm initially free-rides on the low-cost firm's effort, the smaller firm H's market share, the greater the extent of free riding, and the larger the level of cost homogeneity required for the bargaining problem to be non-degenerate.*

That is, for firms to agree on some effective standard, it is imperative that they not be too dissimilar in all respects. If their size difference is big, then the cost differential must be small in order for them to find a common ground.

Provided cost heterogeneity is not too large, firm H is willing to agree to some standards. The highest standard which it is willing to accept is given by

$$z_2 = \frac{1 + \sqrt{1 - \alpha^2 - 2\left(1 - \alpha\right)^2 \frac{\theta_H}{\theta_L}}}{\theta_H}. \tag{34}$$

Firm L's most optimistic expectation of profit corresponds to its favorite standard of all, $1/\theta_L$, or z_2, whichever is smaller. Under large cost homogeneity, defined as

$$\theta_L/\theta_H \geq 1/\left[\alpha(2-\alpha) + \alpha\sqrt{\alpha^2 - 4\alpha + 3}\right], \tag{35}$$

the former is larger.

Therefore, we still have the three regions identified in Section 4 and the results carry over.

6 Conclusion

We have studied a simple model in which consumers cannot observe the quality (or quality effort) choices made by two producers but demand depends instead upon the average quality of the goods available in the market. Before specifying the quality aspect of their products, the two firms engage into bargaining over the adoption of a strictly and costlessly enforced minimum quality (effort) standard. The Kalai-Smorodinsky solution is assumed to capture the outcome of this negotiation.

In the absence of a standard, firms underprovide quality as a result of a classical public good problem. We have shown that if firms have very different costs for quality, then they cannot agree on any common standard, except perhaps a completely ineffective one. When firms are not too dissimilar, then the KS solution selects a standard that is always lower than the joint-profit maximizing, or for that matter the (second-most) efficient, level. The adopted standard always lies closer to the high-cost producer's favorite choice than to the low-cost producer's. Thus, there is a tendency for a duopoly deciding about the minimal level of quality to be provided in a particular industry to set it too low. This somewhat contrasts with Leland (1979)'s finding but it needs to be noted that by prohibiting side transfers and fixing quantities, we have in effect remove any possibility for the industry to use the standard so as to restrict output.

In our model, the adopted standard often decreases in the cost of providing quality of any firm. Nonetheless, when firms' costs are such that the high-cost producer can agree only to a small range of possible norms, the equilibrium standard increases with the low-cost producer's cost as the sudden expansion of the interval of norms to which the inefficient producer is amenable enhances the low-cost producer's bargaining position.

The question of the robustness and generality of these results immediately arises. One may want to get rid of the fixed-quantity assumption but the introduction of a quantity choice considerably complicates the analysis. The extension to larger oligopolies seems more promising. The study of such a "grand bargaining" could be the first step in the study of a more general partition-game, or club-formation, model in which firms "decide" which producers to join to create a "label" or "brand". Of course, such a model will necessitate a specification of the demand-side less rudimentary than the one in this article and awaits future research.

A Appendix

A.1 Intermediate homogeneity case

The equation defining the agreed-upon standard is as follows:

$$\frac{\frac{1}{2}\left[m-\frac{1}{2}\theta_L m^2+a\right]-\frac{1}{2}\left[\frac{1}{4\theta_H}+\frac{1}{2}\frac{1}{4\theta_L}+a\right]}{\frac{1}{2}\left[m-\frac{1}{2}\theta_H m^2+a\right]-\frac{1}{2}\left[\frac{1}{2}\frac{1}{4\theta_H}+\frac{1}{4\theta_L}+a\right]}=\frac{\frac{1}{2}\left[z_2-\frac{1}{2}\theta_L(z_2)^2+a\right]-\frac{1}{2}\left[\frac{1}{4\theta_H}+\frac{1}{2}\frac{1}{4\theta_L}+a\right]}{\frac{1}{2}\left[\frac{1}{\theta_H}-\frac{1}{2}\theta_H\left(\frac{1}{\theta_H}\right)^2+a\right]-\frac{1}{2}\left[\frac{1}{2}\frac{1}{4\theta_H}+\frac{1}{4\theta_L}+a\right]} \tag{36}$$

A.2 Proofs

Proof of Lemma 1 Let $\gamma : \mathbb{R}_+ \to \mathbb{R}^2$ be the function that to each $m \geq 0$ associates the profit gain vector:

$$\gamma(m) \equiv (\gamma_H(m), \gamma_L(m)) \equiv \left(\pi_H \left[x_H^*(m), x_L^*(m)\right] - \pi_H^U, \quad \pi_L \left[x_H^*(m), x_L^*(m)\right] - \pi_L^U\right). \tag{37}$$

Then by definition the feasible set, F, of gains is the range of γ.[17] Since the intersection of F with $\mathbb{R}_+^2$ is what really matters, one looks for the best outcome for firms in the positive orthant only.

It was argued in Section 4.1 that if condition (18) was not satisfied, then firm H's profit could never be greater than when the standard was not binding. That is: if $\frac{\theta_L}{\theta_H} < \frac{2}{3}$, then $\gamma_H(m) < \gamma_H(0)$ for all $m > \frac{1}{2\theta_H}$. That takes care of the small homogeneity case.

Suppose now that condition (18) is satisfied. We have to prove that

$$\text{if } \frac{2}{3} < \frac{\theta_L}{\theta_H} < \frac{4}{3+\sqrt{5}}, \text{ then } \begin{cases} \arg\max\limits_{m \in \gamma^{-1}(\mathbb{R}_+^2)} \gamma_H(m) = \frac{1}{\theta_H} \\ \arg\max\limits_{m \in \gamma^{-1}(\mathbb{R}_+^2)} \gamma_L(m) = z_2 = \frac{1+\sqrt{\frac{3}{4} - \frac{1}{2}\frac{\theta_H}{\theta_L}}}{\theta_H} \end{cases};$$

$$\text{if } \frac{4}{3+\sqrt{5}} \leq \frac{\theta_L}{\theta_H} \leq 1, \text{ then } \begin{cases} \arg\max\limits_{m \in \gamma^{-1}(\mathbb{R}_+^2)} \gamma_H(m) = \frac{1}{\theta_H} \\ \arg\max\limits_{m \in \gamma^{-1}(\mathbb{R}_+^2)} \gamma_L(m) = \frac{1}{\theta_L} \end{cases}.$$

Since condition (18) is satisfied, $\gamma_H(m)$ reaches its maximum at $m = \frac{1}{\theta_H}$. Since $\theta_L \leq \theta_H$ and $\frac{1}{\theta_H} > \frac{1}{2\theta_L}$, we necessarily have $\gamma_L(\frac{1}{\theta_H}) \geq \gamma_H(\frac{1}{\theta_H}) > 0$ and so $\gamma(\frac{1}{\theta_H}) \in \mathbb{R}_+^2$. Hence, we always have

$$\arg\max_{m \in \gamma^{-1}(\mathbb{R}_+^2)} \gamma_H(m) = \frac{1}{\theta_H}. \tag{38}$$

From the analysis in Section 4.1, we know that the unrestricted maximizer of γ_L is $\frac{1}{\theta_L}$. The range of standards $\gamma^{-1}(\mathbb{R}_+^2)$ is $\{m \mid \gamma_H(m) \geq 0\}$, as $\gamma_L(m) \geq$

[17] As a matter of fact, the KS solution is usually defined on convex feasible sets. Technically speaking, we take F to be the convex and comprehensive hull of the range of γ. See Thomson (1994) for an exact definition. Convexity is obtained by assuming that the negotiators can always randomize between outcomes. Comprehensiveness corresponds to a "free-disposal-of-profit assumption" that is innocuous in our context.

$\gamma_H(m)$ for $m \geq \frac{1}{2\theta_H}$ and π_L^U as well as π_H^U are positive. It is found by computing the roots z_1 and z_2 of the following equation:

$$\frac{1}{2}\left[z - \frac{1}{2}\theta_H z^2 + a\right] - \frac{1}{2}\left[\frac{1}{2}\frac{1}{4\theta_H} + \frac{1}{4\theta_L} + a\right] = 0, \tag{39}$$

which equates firm H's profit from the imposition of a binding standard[18] to firm H's profit in the absence of a standard. The discriminant of that equation is $\frac{3}{4} - \frac{1}{2}\frac{\theta_H}{\theta_L}$, which is strictly positive if condition (18) above is met. So we have:

$$z_1 = \frac{1 - \sqrt{\frac{3}{4} - \frac{1}{2}\frac{\theta_H}{\theta_L}}}{\theta_H} \tag{40}$$

$$z_2 = \frac{1 + \sqrt{\frac{3}{4} - \frac{1}{2}\frac{\theta_H}{\theta_L}}}{\theta_H}. \tag{41}$$

and[19] $\gamma^{-1}(\mathbb{R}_+^2) = (z_1, z_2)$. It is obvious that $z_1 < \frac{1}{\theta_H}$. The question of the comparison between z_2 and $\frac{1}{\theta_L}$ can be settled by studying the equation

$$\frac{1 + \sqrt{\frac{3}{4} - \frac{1}{2}\frac{\theta_H}{\theta_L}}}{\theta_H} = \frac{1}{\theta_L} \tag{42}$$

or, equivalently,

$$1 + \sqrt{\frac{3}{4} - \frac{1}{2}z} = z, \tag{43}$$

where $z = \frac{\theta_H}{\theta_L}$. The roots are given by $\frac{3}{4} \pm \frac{\sqrt{5}}{4}$. Thus, if $1 \leq \frac{\theta_H}{\theta_L} \leq \frac{3}{4} + \frac{\sqrt{5}}{4}$, then

$$\arg \max_{m \in \gamma^{-1}(\mathbb{R}_+^2)} \gamma_L(m) = \frac{1}{\theta_L}. \tag{44}$$

[18] That is: binding for *both* firms. We saw that Firm 1's profit is less than its guaranteed profit under no standard if the standard does not affect Firm 2's behavior.

[19] The range of standards for which there are mutual gains and the interests of the two bargainers diverge (that is, corresponding to the downward-sloping section of the feasible set frontier) is $\left[\frac{1}{\theta_H}, \min(z_2, \frac{1}{\theta_L})\right]$. The interval $\left[\frac{1}{2\theta_L}, \frac{1}{\theta_H}\right]$ corresponds to an upward-sloping section of the feasible set frontier as both firms' profits are going up with m. Similarly, the interval $[\frac{1}{\theta_L}, +\infty)$ also corresponds to an upward-sloping section of the profit possibility set as both firms' profits are going down with m. Over these two latter intervals, the interests of the two bargainers can be said to be aligned.

If $\frac{3}{4} + \frac{\sqrt{5}}{4} < \frac{\theta_H}{\theta_L} < \frac{3}{2}$, then

$$\arg \max_{m \in \gamma^{-1}(\mathbb{R}_+^2)} \gamma_L(m) = z_2 = \frac{1 + \sqrt{\frac{3}{4} - \frac{1}{2}\frac{\theta_H}{\theta_L}}}{\theta_H}. \tag{45}$$

End of proof.

Proof of Proposition 3 At the KS solution, the following equation is verified:

$$\frac{\frac{1}{2}\left[m - \frac{1}{2}\theta_L m^2 + a\right] - \frac{1}{2}\left[\frac{1}{4\theta_H} + \frac{1}{2}\frac{1}{4\theta_L} + a\right]}{\frac{1}{2}\left[m - \frac{1}{2}\theta_H m^2 + a\right] - \frac{1}{2}\left[\frac{1}{2}\frac{1}{4\theta_H} + \frac{1}{4\theta_L} + a\right]} = \frac{\frac{1}{2}\left[\frac{1}{\theta_L} - \frac{1}{2}\theta_L\left(\frac{1}{\theta_L}\right)^2 + a\right] - \frac{1}{2}\left[\frac{1}{4\theta_H} + \frac{1}{2}\frac{1}{4\theta_L} + a\right]}{\frac{1}{2}\left[\frac{1}{\theta_H} - \frac{1}{2}\theta_H\left(\frac{1}{\theta_H}\right)^2 + a\right] - \frac{1}{2}\left[\frac{1}{2}\frac{1}{4\theta_H} + \frac{1}{4\theta_L} + a\right]} \tag{46}$$

It can be rewritten

$$-\frac{3\left(\theta_L - \theta_H\right)}{16\theta_H\theta_L}m^2 + \frac{5\left(\theta_L - \theta_H\right)}{8\theta_H\theta_L}m + \frac{1}{8}\left(\frac{1}{\theta_L} - \frac{1}{\theta_H}\right) = 0 \tag{47}$$

or

$$-\frac{3\left(\theta_L + \theta_H\right)}{2}m^2 + 5m - \frac{\theta_L + \theta_H}{\theta_H\theta_L} = 0 \tag{48}$$

The discriminant of this equation is

$$\Delta = 25 - 6\frac{\left(\theta_L + \theta_H\right)^2}{\theta_H\theta_L}, \tag{49}$$

which is positive for $\theta_H \leq \frac{3}{2}\theta_L$.

Solving for m:

$$m = \frac{5 \pm \sqrt{25 - 6\frac{(\theta_H + \theta_L)^2}{\theta_H\theta_L}}}{3(\theta_H + \theta_L)} \tag{50}$$

Both roots are positive but it can be shown that only the bigger one lies in $\left[\frac{1}{\theta_H}, \frac{1}{\theta_L}\right]$, as the dashed line standing for the ratio of maximal profit increments on Figure 3 always crosses the upward-sloping section of the feasible set before crossing its downward-sloping section. Thus we have that in equilibrium:

$$\hat{m} = \frac{5 + \sqrt{25 - 6\frac{(\theta_H+\theta_L)^2}{\theta_H\theta_L}}}{3(\theta_H + \theta_L)} \tag{51}$$

END OF PROOF.

Proof of Proposition 4 (i) Observe that

$$\frac{\partial}{\partial\theta_H}\left[\frac{(\theta_H + \theta_L)^2}{\theta_H\theta_L}\right] = \frac{\theta_H(\theta_H + \theta_L)}{(\theta_H\theta_L)^2}\left(\theta_H - \theta_L\right), \tag{52}$$

which is positive for any values of θ_H and θ_L since by assumption $\theta_H > \theta_L$. Thus, when θ_H goes up, the numerator in $\hat{m}$ goes down. Since the denominator goes up, $\hat{m}$ is bound to decrease.

(ii) Observe that

$$\frac{\partial\hat{m}}{\partial\theta_L} = \frac{\frac{3\theta_H(\theta_H-\theta_L)(\theta_H+\theta_L)^2}{(\theta_H\theta_L)^2\sqrt{25-6\frac{(\theta_H+\theta_L)^2}{\theta_H\theta_L}}} - \left(5 + \sqrt{25 - 6\frac{(\theta_H+\theta_L)^2}{\theta_H\theta_L}}\right)}{3(\theta_H + \theta_L)^2}. \tag{53}$$

To know the sign of this derivative, it is sufficient to study the numerator. If we can show that

$$\frac{3\theta_H\left(\theta_H - \theta_L\right)\left(\theta_H + \theta_L\right)^2}{(\theta_H\theta_L)^2\sqrt{25 - 6\frac{(\theta_H+\theta_L)^2}{\theta_H\theta_L}}} < \sqrt{25 - 6\frac{(\theta_H + \theta_L)^2}{\theta_H\theta_L}}, \tag{54}$$

then we are done. Manipulating this inequality, one gets

$$\frac{\theta_H(\theta_H + \theta_L)^3}{(\theta_H\theta_L)^2} < \frac{25}{3}. \tag{55}$$

Observe that the left-hand side is strictly increasing in θ_H. By assumption $\theta_H > \theta_L$, so

$$\frac{\theta_H(\theta_H + \theta_L)^3}{(\theta_H\theta_L)^2} < \frac{\theta_H(2\theta_H)^3}{(\theta_H)^2} = \frac{8(\theta_H)^4}{(\theta_H)^4}. \tag{56}$$

The right-hand side of this inequality is of course smaller than $\frac{25}{3}$. Hence, $\frac{\partial\hat{m}}{\partial\theta_L} < 0$ for any θ_H.

END OF PROOF.

Proof of Proposition 7 Suppose that, for a given pair (θ_H, θ_L) such that $\frac{3}{4} + \frac{\sqrt{5}}{4} < \frac{\theta_H}{\theta_L} < \frac{3}{2}$, Equation (36) holds for a certain $\tilde{m}$, taken to be fixed in what follows.

Suppose that θ_H increases infinitesimally. D will go down as it is a positive linear function of $\frac{1}{\theta_H}$. B will decrease by still a bigger percentage for initially, Firm 1's actual profit increment is less than its maximum profit increment, and the change in Firm 1's cost brought about by the increase in θ_H is proportional to the prevailing effort level, $\tilde{m}$, which is bigger than $\frac{1}{\theta_H}$.

A will go up following the decrease in Firm 2's guaranteed profit. (With higher cost, Firm 1 free-rides even more in the absence of a standard.) Without careful calculations, it is not possible to tell whether C will increase or decrease. On one hand, Firm 2's guaranteed profit goes up but on the other hand, its maximal profit $z_2 - \frac{1}{2}\theta_L(z_2)^2$ goes down, as $\partial z_2/\partial\theta_H$ is negative. Yet, from this extra-impact on Firm 2's maximal profit (and from the fact that initially Firm 2's actual profit increment is less than its maximum profit increment), it is clear that in any case C grows by a strictly smaller percentage than A.

Thus, for a fixed $\tilde{m}$, we have that A/B grows by a strictly greater percentage than C/D. Therefore, $\tilde{m}$ must have decreased in order to preserve Equation (36).

End of proof.

Proof of Proposition 8 Suppose that, for a given pair (θ_H, θ_L) such that $\frac{3}{4} + \frac{\sqrt{5}}{4} < \frac{\theta_H}{\theta_L} < \frac{3}{2}$, Equation (36) holds for a certain $\tilde{m}$, taken to be fixed in what follows.

Suppose that θ_L increases infinitesimally. Firm 1's maximal profit increment, D, goes up as its guaranteed profit goes down and its maximal profit remains unchanged. Firm 1's actual profit increment, B, goes up by a higher percentage as the change in guaranteed profit is the same but applies to a smaller initial value. Firm 2's actual gain, A, goes down. Indeed, one has

$$\frac{dA}{d\theta_L} = \frac{1}{2}\left[\left(\frac{1}{2\theta_L}\right)^2 - (\tilde{m})^2\right] \tag{57}$$

and $\tilde{m}$ is of course greater than $\frac{1}{2\theta_L}$. So the left-hand side of Equation (36), A/B, goes down.

If it can be shown that the right-hand side goes up, then there will be no doubt that the standard has to increase in order for Equality (36) to remain satisfied. Observe that

$$\frac{dC}{d\theta_L} = \frac{1}{8(\theta_L)^2} - \frac{1}{2}(z_2)^2 + (1 - \theta_L z_2)\frac{dz_2}{d\theta_L} \tag{58}$$

and

$$\frac{dz_2}{d\theta_L} = \frac{1}{4\left(\theta_L\right)^2 \sqrt{\frac{3}{4} - \frac{1}{2}\frac{\theta_H}{\theta_L}}} \tag{59}$$

It is readily seen that

$$\lim_{\theta_L \to \frac{2}{3}\theta_H} \frac{dC}{d\theta_L} = +\infty \tag{60}$$

By continuity, $dC/d\theta_L$ is very large in the neighborhood of $\frac{2}{3}\theta_H$. Thus, C/D goes up in this neighborhood, provided it is sufficiently small. As a result, $\tilde{m}$ needs to increase in order for Equation (36) to be verified again.

End of proof.

References

[1] Akerlof, George A. (1970). "The Market for 'Lemons': Quality Uncertainty and the Market Mechanism." *Quarterly Journal of Economics*, 84, 488-500.

[2] Argenton, Cédric (2006). "The market for melons: Cournot competition with unobservable qualities." SSE/EFI Working Paper No. 618.

[3] Crampes, Claude and Abraham Hollander (1995). "Duopoly and Quality Standards." *European Economic Review*, 39, 71-82.

[4] Ecchia, Giulio and Luca Lambertini (1997). "Minimum quality standards and collusion." *Journal of Industrial Economics*, 45, 101-113.

[5] Kalai, Ehud and Robert W. Rosenthal (1978), "Arbitration of Two-Party Disputes Under Ignorance." *International Journal of Game Theory*, 7, 65-72.

[6] Kalai, Ehud and Meir Smorodinsky (1975). "Other Solutions to Nash's Bargaining Problem." *Econometrica*, 47, 513-518.

[7] Kessel, Reuben (1958). "Price discrimination in medicine." *Journal of Law and Economics*, 1,20-53.

[8] Leland, Hayne E. (1979). "Quacks, Lemons, and Licensing: A Theory of Minimum Quality Standards." *Journal of Political Economy*, 87, 1328-1346.

[9] Lerner, Josh and Jean Tirole (2004). "A Model of Forum Shopping, with Special Reference to Standard Setting Organizations." *NBER Working Paper* #10664.

[10] Miyagawa, Eiichi (2002). "Subgame-perfect implementation of bargaining solutions." *Games and Economic Behavior*, 41, 292-308.

[11] Moulin, Hervé (1984). "Implementing the Kalai-Smorodinsky Bargaining Solution." *Journal of Economic Theory*, 33, 32-45.

[12] Mussa, Michael and Sherwin Rosen (1978). "Monopoly and Product Quality." *Journal of Economic Theory*, 18, 301-317.

[13] Ronnen, Uri (1991). "Minimum Quality Standards, Fixed Costs, and Competition." *The Rand Journal of Economics*, 22, 490-504.

[14] Scarpa, Carlo (1998). "Minimum Quality Standards with More than Two Firms." *International Journal of Industrial Organization*, 16, 665-676.

[15] Spence, A. Michael (1975). "Monopoly, Quality, and Regulation." *Bell Journal of Economics*, 6, 417-429.

[16] Thomson, William (1994). "Cooperative Models of Bargaining." In *Handbook of Game Theory*, Vol. 2, edited by Robert J. Aumann and Sergiu Hart. Elsevier Science.

[17] Tirole, Jean (1988). *The theory of industrial organization*. Cambridge, Mass.: MIT Press.

[18] Valletti, Tommaso M. (2000). "Minimum Quality Standards Under Cournot Competition." *Journal of Regulatory Economics*, 18, 235-245.